

2.5 Reason Using Properties from Algebra

Let a , b , and c be real numbers.

Properties of Equality

Addition Property--If $a = b$ and $c = d$, then $a + c = b + d$.

Subtraction Property--If $a = b$ and $c = d$, then $a - c = b - d$.

Multiplication Property--If $a = b$, then $c \cdot a = c \cdot b$.

Division Property-- If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$

Distributive Property-- $a(b + c) = ab + ac$

Substitution Property--If $a = b$, then either a or b may be substituted for the other in any equation.

(Combining like terms)

Reflexive Property-- $a = a$

Symmetric Property--If $a = b$, then $b = a$.

Transitive Property--If $a = b$ and $b = c$, then $a = c$.

Reflexive, symmetric, and transitive also work with **congruence**.

Example 1

- | | |
|----------------------------------|------------------------|
| 1. $8(x - 5) = 32$ | 1. <i>Given</i> |
| 2. $8x - 40 = 32$ | 2. <i>Distributive</i> |
| 3. $8x - 40 + 40 = 32 + 40$ | 3. <i>Addition</i> |
| 4. $8x = 72$ | 4. <i>Substitution</i> |
| 5. $\frac{8x}{8} = \frac{72}{8}$ | 5. <i>Division</i> |
| 6. $x = 9$ | 6. <i>Subst.</i> |

Example 1

- | | |
|--------------------|------------------------|
| 1. $8(x - 5) = 32$ | 1. <i>Given</i> |
| 2. $8x - 40 = 32$ | 2. <i>Distributive</i> |
| 3. $8x = 72$ | 3. <i>Addition</i> |
| 4. $x = 9$ | 4. <i>Division</i> |

Example 2

- | | |
|------------------|--------------------|
| 1. $5x - 3 = 12$ | 1. <i>Given</i> |
| 2. $5x = 15$ | 2. <i>Addition</i> |
| 3. $x = 3$ | 3. <i>Division</i> |

Example 3

- | | |
|-----------------------|------------------|
| 1. $8x + 3x - 9 = 24$ | 1. <i>Given</i> |
| 2. $11x - 9 = 24$ | 2. <i>Subst.</i> |
| 3. $11x = 33$ | 3. <i>Add.</i> |
| 4. $x = 3$ | 4. <i>Div.</i> |

Example 4

- $2(5 - 3x) - 4(x + 7) = 92$
- $10 - 6x - 4x - 28 = 92$
- $-10x - 18 = 92$
- $-10x = 110$
- $x = -11$
- $-11 = x$

- Given
- Dist.
- Substitution
- Add
- Div.
- Symmetric

Justify each statement.

- Reflexive $m\angle 1 = m\angle 1$
- Addition If $m\angle 1 = m\angle 2$, then $m\angle 1 + m\angle 3 = m\angle 2 + m\angle 3$
- Mult. If $AB = CD$, then $2 \cdot AB = 2 \cdot CD$.
- Symmetric If $RS = XY$, then $XY = RS$
- Transitive If $m\angle A = m\angle B$, and $m\angle B = m\angle C$, then $m\angle A = m\angle C$
- division If $2 \cdot m\angle 1 = 90$, then $m\angle 1 = 45$
- Subst. If $m\angle 9 + m\angle 10 = 150^\circ$, and the $m\angle 10 = 48^\circ$, then $m\angle 9 + 48 = 150$.
- Subtr. If $m\angle 9 + 48 = 150$, then $m\angle 9 = 102$.

State the property that justifies each statement.

- If $2x = 5$, then $x = \frac{5}{2}$ Div.
- If $\frac{x}{2} = 7$, then $x = 14$. Mult.
- If $x = 5$ and $b = 5$, then $x = b$. Subst. (Not transitive b/c 8 order)
- If $XY - AB = WZ - AB$, then $XY = WZ$. Add.

8. Solve $\frac{1}{2}x + 4x - 7 = 11$. List the property that justifies each step.

9. Complete the following proof.

Given: $5 - \frac{2}{3}x = 1$

Prove: $x = 6$

Proof:

Statements

a. ?

b. $3(5 - \frac{2}{3}x) = 3(1)$

c. $15 - 2x = 3$

d. ?

e. $x = 6$

Reasons

a. Given

b. ? Mult.c. ? distr./subst

d. Subtraction Prop.

e. ? Div.

Hw
p108-109
3-7, 9, 21-25,

WRITING REASONS Copy the logical argument. Write a reason for each step.

- | | |
|-----------------------------|-------------------------------|
| 3. $3x - 12 = 7x + 8$ Given | 4. $5(x - 1) = 4x + 13$ Given |
| $-4x - 12 = 8$? | $5x - 5 = 4x + 13$? |
| $-4x = 20$? | $x - 5 = 13$? |
| $x = -5$? | $x = 18$? |

5. ★ **MULTIPLE CHOICE** Name the property of equality the statement illustrates: If $XY = AB$ and $AB = GH$, then $XY = GH$.

- (A) Substitution (B) Reflexive (C) Symmetric (D) Transitive

WRITING REASONS Solve the equation. Write a reason for each step.

- | | |
|---------------------|-----------------------|
| 6. $5x - 10 = -40$ | 7. $4x + 9 = 16 - 3x$ |
| 9. $3(2x + 11) = 9$ | |

COMPLETING STATEMENTS In Exercises 21–25, use the property to copy and complete the statement.

- (21.) Substitution Property of Equality: If $AB = 20$, then $AB + CD = ?$.
- (22.) Symmetric Property of Equality: If $m\angle 1 = m\angle 2$, then $?$.
- (23.) Addition Property of Equality: If $AB = CD$, then $? + EF = ? + EF$.
- (24.) Distributive Property: If $5(x + 8) = 2$, then $?x + ? = 2$.
- (25.) Transitive Property of Equality: If $m\angle 1 = m\angle 2$ and $m\angle 2 = m\angle 3$, then $?$.